

Advanced Algorithms
WS 2019
Homework Assignment 7

Problem 18:

Consider an algorithm for an optimization problem that can reduce an instance of that problem of size k to an instance of size $k - 1$ but does a mistake (in a sense that it loses optimality) with probability $1/k$. Once the problem is reduced to an instance of size 1, the problem can be solved optimally (which would be a globally optimal solution if optimality has not been lost before).

- (a) Suppose that it can be determined efficiently whether the algorithm does a mistake in a reduction so that a check after each reduction is feasible. How many repetitions of the algorithm are needed on expectation, over all reductions, until an optimal result has been found for an instance of size n ?
- (b) Suppose that an error in the reduction cannot be determined efficiently (i.e., one can only discover an error at the very end). Propose an algorithm for this case that is similar to FastCut. What is the error probability of your algorithm? What would be its runtime if a reduction from k to $k - 1$ requires a runtime of $O(k)$?

Problem 19:

Show the following results with the help of Bayes' formula:

- (a) For any collection of events A, B, C ,

$$\Pr[A \mid B \cap C] = \frac{\Pr[A \cap B \mid C]}{\Pr[B \mid C]}$$

- (b) For any events A and B it holds: If $\Pr[A \mid B] = \Pr[A]$ then $\Pr[A \mid \bar{B}] = \Pr[A]$.
- (c) For any collection of events A_1, \dots, A_n ,

$$\Pr \left[\bigcap_{i=1}^n A_i \right] = \prod_{i=1}^n \Pr[A_i \mid A_1 \cap \dots \cap A_{i-1}]$$