Advanced Distributed Algorithms and Data Structures
SS 2019
Homework Assignment 5

Problem 1:
Prove Corollay 4.20 on slide 125 of Chapter 4.
Hint: Use the fact that an expander graph has a constant expansion.

Problem 2:
Implement the algorithm shown on slide 127 of Chapter 4 to transform a tree into a cycle using a parallel form of DFS in the NetSimLan environment.

Problem 3:

(a) Prove the Markov inequality on slide 11 of Chapter 5.

(b) Prove the Chebychev inequality on slide 11 of Chapter 5.

Problem 4:
Consider slide 151 of Chapter 4. Suppose that the nodes pick their endpoints uniformly and independently at random among the nodes of the cycle. (I.e., in the example, 4,2, and 6 pick node 2 as their endpoint.) Show the following two statements with the help of the Chernoff bounds and the union bound:

(a) With high probability, at most $O(\log n)$ nodes choose a node as their endpoint.

(b) With high probability, there are at most $O(\log n)$ consecutive nodes along the cycle that are not picked as an endpoint by any node.

Hint: First, prove statement (a) for a fixed node and statement (b) for a fixed consecutive sequence of nodes using the Chernoff bounds, and then use the union bound to show that this can be genelized to any node in (a) and any consecutive sequence of nodes in (b).
The union bound for a collection of events $A_1, \ldots, A_n$ states that

$$\Pr[A_1 \cup A_2 \cup \ldots \cup A_n] \leq \sum_{i=1}^{n} \Pr[A_i]$$