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Randomized Algorithms

SS 2018

Homework Assignment 7

Problem 19:
Consider an algorithm for an optimization problem that can reduce an instance of that problem of size k to an instance of size $k-1$ but does a mistake (in a sense that it loses optimality) with probability $1/k$. Once the problem is reduced to an instance of size 1, the problem can be solved optimally (which would be a globally optimal solution if optimality has not been lost before).

(a) Suppose that it can be determined efficiently whether the algorithm does a mistake in a reduction so that a check after each reduction is feasible. How many repetitions of the algorithm are needed on expectation, over all reductions, until an optimal result has been found for an instance of size n?

(b) Suppose that an error in the reduction cannot be determined efficiently (i.e., one can only discover an error at the very end). Propose an algorithm for this case that is similar to FastCut. What is the error probability of your algorithm? What would be its runtime if a reduction from k to $k-1$ requires a runtime of $O(k)$?

Problem 20:

(a) Show with the help of Bayes formula that for any collection of events A, B, C,

$$\Pr[A \mid B \cap C] = \frac{\Pr[A \cap B \mid C]}{\Pr[B \mid C]}$$

(b) Show with the help of Bayes formula that for any collection of events $A_1, \ldots, A_n$,

$$\Pr\left[\bigcap_{i=1}^{n} A_i\right] = \prod_{i=1}^{n} \Pr[A_i \mid A_1 \cap \ldots \cap A_{i-1}]$$

Problem 21:
Prove Lemma 5.7 and apply it to the following problem. Show that for any graph $G = (V, E)$ of maximum degree d the nodes of G can be colored with $\lceil 2e \cdot d \rceil$ colors so that no two neighboring nodes have the same color.