Decision Tree and Instance-Based Learning for Label Ranking

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Label Ranking (an example)

Learning customers' preferences on cars:

	label ranking
customer 1	MINI $\succ$ Toyota $\succ$ BMW
customer 2	BMW $\succ$ MINI $\succ$ Toyota
customer 3	BMW $\succ$ Toyota $\succ$ MINI
customer 4	Toyota $\succ$ MINI $\succ$ BMW
new customer	???

where the customers could be described by feature vectors, e.g., (gender, age, place of birth, has child, ...)

Label Ranking (an example)

Learning customers' preferences on cars:

	MINI	Toyota	BMW
customer 1	1	2	3
customer 2	2	3	1
customer 3	3	2	1
customer 4	2	1	3
new customer	?	?	?

$\pi(i)$ = position of the i-th label in the ranking

1: MINI 2: Toyota 3: BMW

Label Ranking (more formally)

Given:

- a set of training instances $\{\mathbf{x}_k \mid k = 1 \ldots m\} \subseteq \mathbf{X}$
- a set of labels $L = \{l_i \mid i = 1 \ldots n\}$
- for each training instance $\mathbf{x}_k$: a set of *pairwise preferences* of the form $l_i \succ_{\mathbf{x}_k} l_j$ (for some of the labels)

Find:

- A ranking function ($\mathcal{X} \rightarrow \Omega$ mapping) that maps each $\mathbf{x} \in \mathbf{X}$ to a ranking $\succ_{\mathbf{x}}$ of L (permutation $\pi_{\mathbf{x}}$) and generalizes well in terms of a loss function on rankings (e.g., *Kendall's tau*)

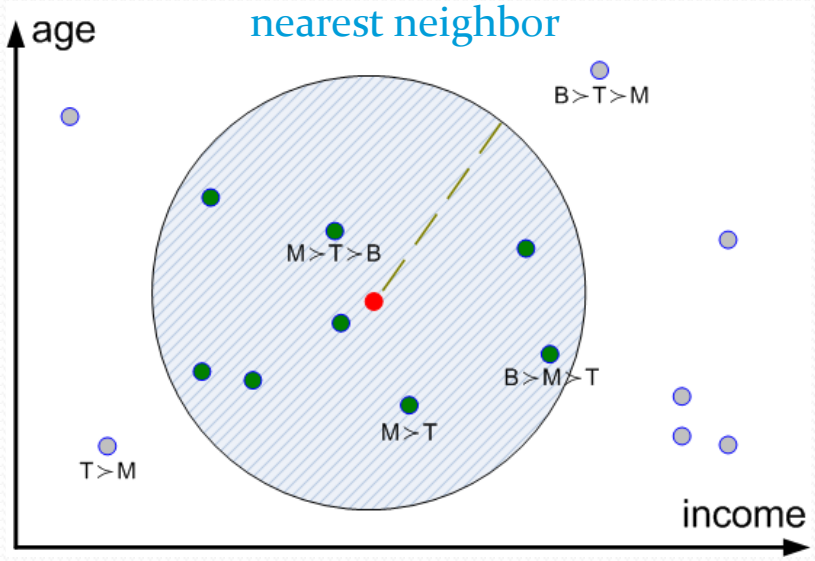
Existing Approaches

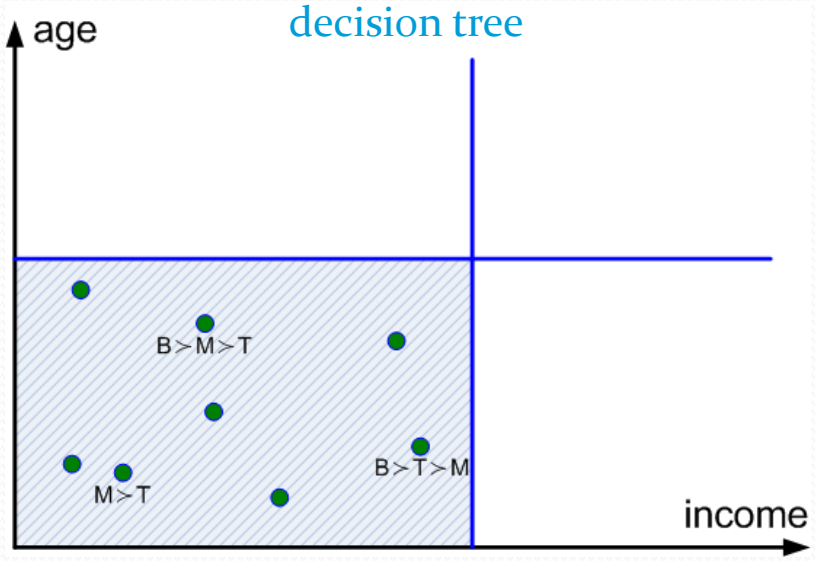
… essentially reduce label ranking to classification:

- Ranking by pairwise comparison
  Fürnkranz and Hüllermeier, ECML-03
- Constraint classification (CC)
  Har-Peled , Roth and Zimak, NIPS-03
- Log linear models for label ranking
  Dekel, Manning and Singer, NIPS-03

  – are efficient but may come with a loss of information
  – may have an improper bias and lack flexibility
  – may produce models that are not easily interpretable

Local Approach (this work)





- Target function $\mathcal{X} \rightarrow \Omega$ is estimated (on demand) in a local way.
- Distribution of rankings is (approx.) constant in a local region.
- Core part is to estimate the locally constant model.

Local Approach (this work)

- Output (ranking) of an instance x is generated according to a distribution $\mathcal{P}(\cdot \mid \mathbf{x})$ on Ω.

- This distribution is (approximately) constant within the local region under consideration.

- Nearby preferences are considered as a sample generated by $\mathcal{P}$, which is estimated on the basis of this sample via ML.

Probabilistic Model for Ranking

Mallows model (Mallows, Biometrika, 1957)

$$\mathcal{P}(\sigma|\theta,\pi) = \frac{\exp(-\theta d(\pi,\sigma))}{\phi(\theta,\pi)}$$

with

center ranking $\pi \in \Omega$

spread parameter $\theta > 0$

and $d(\cdot)$ is a right invariant metric on permutations

$$\forall \pi, \sigma, \nu \in \Omega,\ d(\pi,\sigma) = d(\pi\nu, \sigma\nu).$$

Inference (complete rankings)

Rankings $\boldsymbol{\sigma} = \{\sigma_1, \ldots, \sigma_k\}$ observed locally.

$$\begin{aligned}
\mathcal{P}(\boldsymbol{\sigma}|\theta,\pi) &= \prod_{i=1}^{k} \mathcal{P}(\sigma_i|\theta,\pi) \\
&= \prod_{i=1}^{k} \frac{\exp\left(-\theta d(\sigma_i,\pi)\right)}{\phi(\theta)} \\
&= \frac{\exp\left(-\theta(d(\sigma_1,\pi)+\ldots+d(\sigma_k,\pi))\right)}{\phi^k(\theta)} \\
&= \frac{\exp\left(-\theta\sum_{i=1}^{k} d(\sigma_i,\pi)\right)}{\left(\prod_{j=1}^{n}\frac{1-\exp(-j\theta)}{1-\exp(-\theta)}\right)^k}.
\end{aligned}$$

ML

$$\hat{\pi} = \arg\min_{\pi} \sum_{i=1}^{k} d(\sigma_i,\pi)$$

monotone in θ

$$\frac{1}{k}\sum_{i=1}^{k} d(\sigma_i,\hat{\pi}) = \frac{n\exp(-\theta)}{1-\exp(-\theta)} - \sum_{j=1}^{n}\frac{j\exp(-j\theta)}{1-\exp(-j\theta)}$$

Inference (incomplete rankings)

Probability of an incomplete ranking:

$$\mathcal{P}(E(\sigma_i) \mid \theta, \pi) = \sum_{\sigma \in E(\sigma_i)} \mathcal{P}(\sigma \mid \theta, \pi)$$

where $E(\sigma_i)$ denotes the set of consistent extensions of T.

Example for label set $\{a,b,c\}$:

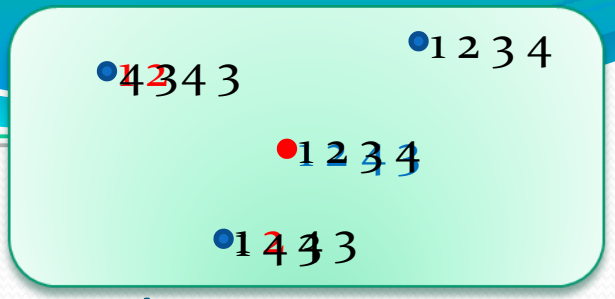
Observation σ	Extensions $E(\sigma)$
$a \succ b$	$a \succ b \succ c$ $a \succ c \succ b$ $c \succ a \succ b$

Inference (incomplete rankings) cont.

The corresponding likelihood:

$$\begin{aligned}\mathcal{P}(\boldsymbol{\sigma}|\theta,\pi) &= \prod_{i=1}^{k}\mathcal{P}(E(\sigma_i)|\theta,\pi)\\ &= \prod_{i=1}^{k}\sum_{\sigma\in E(\sigma_i)}\mathcal{P}(\sigma|\theta,\pi)\\ &= \frac{\prod_{i=1}^{k}\sum_{\sigma\in E(\sigma_i)}\exp\left(-\theta d(\sigma,\pi)\right)}{\left(\prod_{j=1}^{n}\frac{1-\exp(-j\theta)}{1-\exp(-\theta)}\right)^{k}}.\end{aligned}$$

Exact MLE $(\hat{\pi},\hat{\theta}) = \arg\max_{\pi,\theta}\mathcal{P}(\boldsymbol{\sigma}|\theta,\pi)$ becomes infeasible when n is large. Approximation is needed.



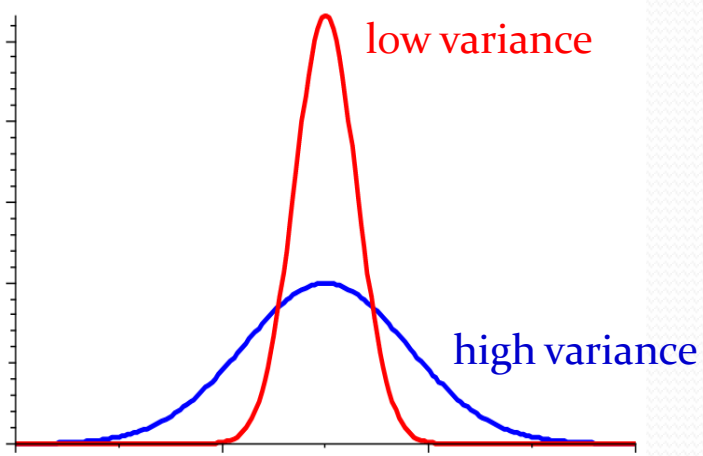
Inference (incomplete rankings) cont.

Approximation via a variant of EM, viewing the non-observed labels as hidden variables.

- replace the E-step of EM algorithm with a maximization step (widely used in learning HMM, K-means clustering, etc.)

1. Start with an initial center ranking (via *generalized Borda count*)
2. Replace an incomplete observation with its most probable extension (*first M-step*, can be done efficiently)
3. Obtain MLE as in the complete ranking case (*second M-step*)
4. Replace the initial center ranking with current estimation
5. Repeat until convergence

Inference

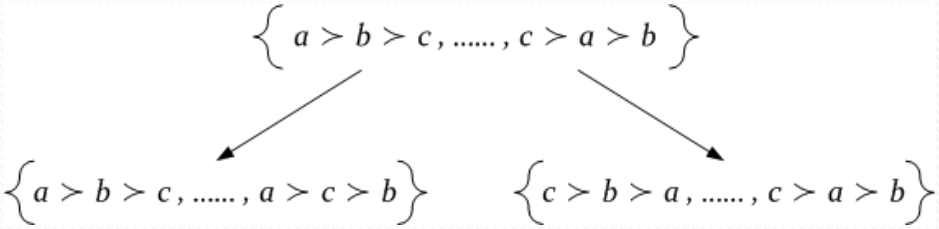


Not only the estimated ranking $\hat{\pi}$ is of interest …

… but also the spread parameter $\hat{\theta}$, which is a measure of precision and, therefore, reflects the confidence/reliability of the prediction (just like the variance of an estimated mean).

The bigger $\hat{\theta}$, the more peaked the distribution around the center ranking.

Label Ranking Trees



Major modifications:

- split criterion

  Split ranking set T into T^+ and T^-, maximizing goodness-of-fit

$$\frac{|T^+| \cdot \theta^+ + |T^-| \cdot \theta^-}{|T|}$$

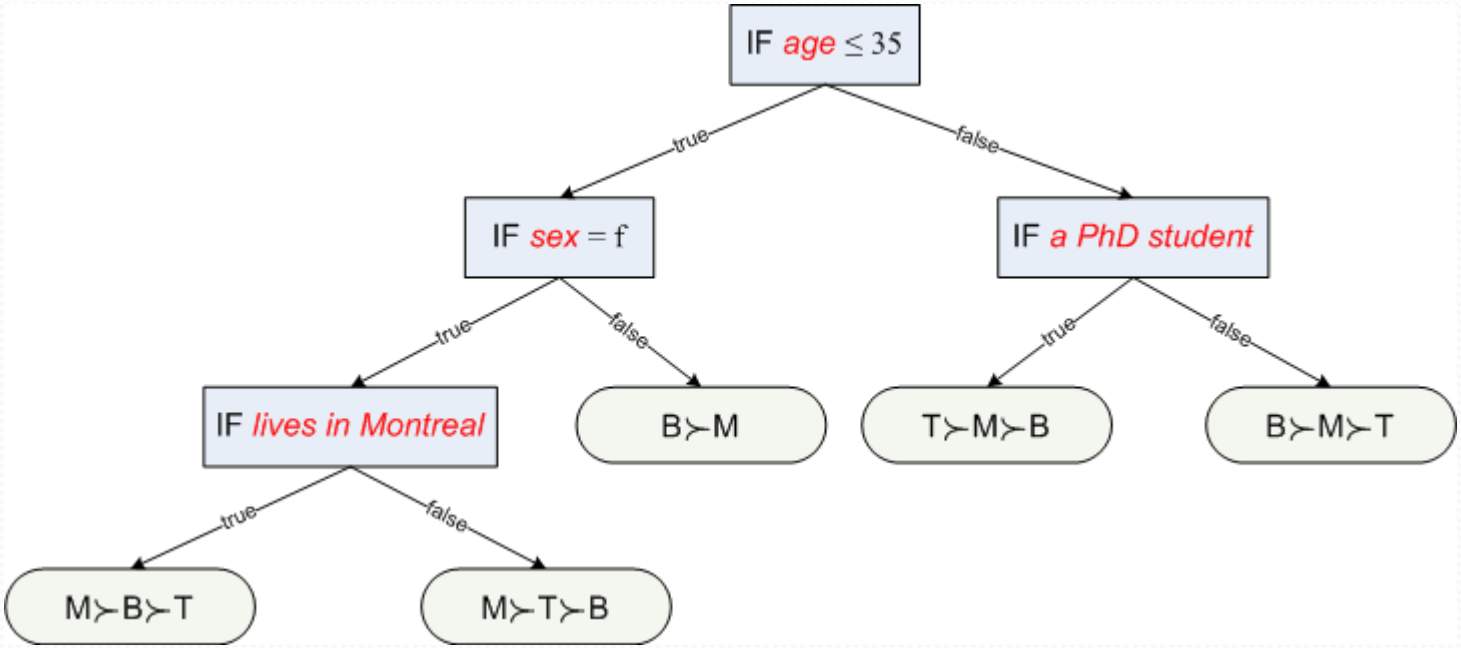
- stopping criterion for partition
  1. tree is pure

     any two labels in two different rankings have the same order
  2. number of labels in a node is too small

     prevent an excessive fragmentation

Label Ranking Trees

Labels: BMW, Mini, Toyota



Experimental Results

	complete rankings			30% missing labels			60% missing labels		
	CC	IBLR	LRT	CC	IBLR	LRT	CC	IBLR	LRT
authorship	.920(2)	.936(1)	.882(3) 1.1	.891(2)	.932(1)	.871(3) 0.9	.835(2)	.920(1)	.828(3) 0.7
bodyfat	.281(1)	.248(2)	.117(3) 1.6	.260(1)	.223(2)	.097(3) 1.7	.224(1)	.180(2)	.070(3) 1.0
calhousing	.250(3)	.351(1)	.324(2) 0.7	.249(3)	.327(1)	.307(2) 0.5	.247(3)	.289(1)	.273(2) 0.3
cpu-small	.475(2)	.506(1)	.447(3) 2.3	.474(2)	.498(1)	.405(3) 2.3	.470(2)	.480(1)	.367(3) 1.5
elevators	.768(1)	.733(3)	.760(2) 0.2	.767(1)	.719(3)	.756(2) 0.2	.765(1)	.690(3)	.742(2) 0.3
fried	.999(1)	.935(2)	.890(3) 5.5	.998(1)	.928(2)	.863(3) 5.3	.997(1)	.895(2)	.809(3) 3.0
glass	.846(3)	.865(2)	.883(1) 2.5	.835(2)	.824(3)	.850(1) 2.0	.789(2)	.771(3)	.799(1) 2.0
housing	.660(3)	.745(2)	.797(1) 2.3	.655(3)	.697(2)	.734(1) 2.4	.638(1)	.630(3)	.634(2) 1.5
iris	.836(3)	.966(1)	.947(2) 1.5	.807(3)	.945(1)	.909(2) 1.2	.743(3)	.882(1)	.794(2) 1.5
pendigits	.903(3)	.944(1)	.935(2) 6.2	.902(3)	.924(1)	.914(2) 3.2	.900(1)	.899(2)	.871(3) 2.2
segment	.914(3)	.959(1)	.949(2) 3.8	.911(3)	.934(1)	.933(2) 3.8	.902(2)	.902(3)	.903(1) 2.3
stock	.737(3)	.927(1)	.895(2) 1.5	.735(3)	.904(1)	.877(2) 1.6	.724(3)	.858(1)	.827(2) 1.1
vehicle	.855(2)	.862(1)	.827(3) 0.8	.839(2)	.842(1)	.819(3) 0.9	.810(1)	.791(2)	.764(3) 0.5
vowel	.623(3)	.900(1)	.794(2) 4.6	.615(3)	.824(1)	.718(2) 3.6	.598(3)	.722(1)	.615(2) 3.2
wine	.933(2)	.949(1)	.882(3) 0.8	.911(2)	.941(1)	.862(3) 1.1	.853(1)	.789(2)	.752(3) 0.8
wisconsin	.629(1)	.506(2)	.343(3) 1.6	.617(1)	.484(2)	.284(3) 1.5	.566(1)	.438(2)	.251(3) 1.6
average rank	2.25	1.44	2.31	2.19	1.50	2.31	1.75	1.88	2.38

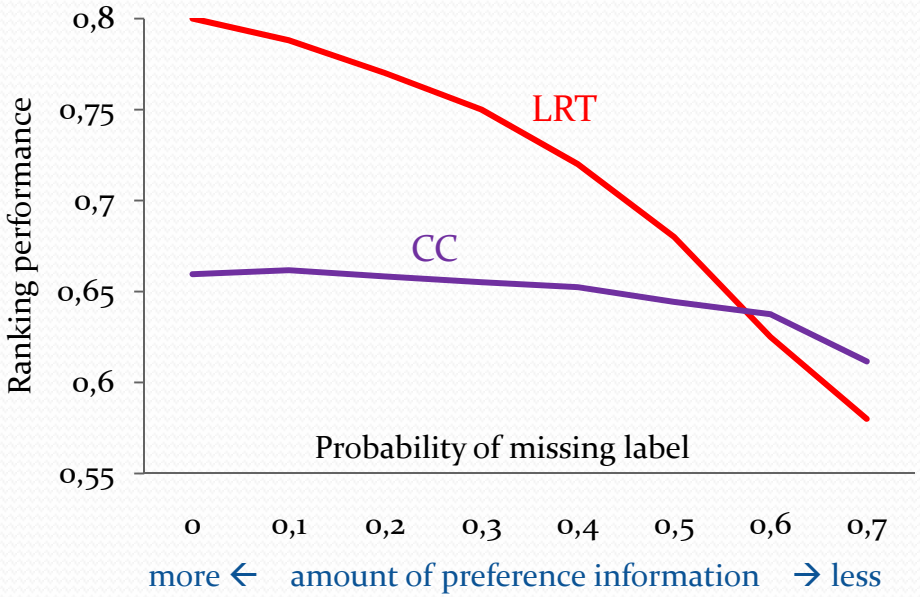
IBLR: instance-based label ranking LRT: label ranking trees

CC: constraint classification

Performance in terms of *Kentall's tau*

Accuracy (Kendall's tau)

Typical "learning curves":



Main observation: Local methods are more flexible and can exploit more preference information compared with the model-based approach.

Take-away Messages

- An instance-based method for label ranking using a probabilistic model.
- Suitable for complete and incomplete rankings.
- Comes with a natural measure of the reliability of a prediction. Makes other types of learners possible: label ranking trees.

- More efficient inference for the incomplete case.
- Dealing with variants of the label ranking problem, such as calibrated label ranking and multi-label classification.

Thanks!

Google “kebi germany” for more info.