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11. July 2016

Complexity Theory

SS 2016

Class Handout 13

Exercise 1:
Show that

$$\mathbf{PH} \subseteq \mathbf{PSPACE} \ .$$

Exercise 2:
Prove that if $\mathbf{PH} = \mathbf{PSPACE}$, then there is a k such that $\Sigma_k = \mathbf{PSPACE}$.

Exercise 3:
Consider the properties of the p-balanced NTM in the Definition 8.3 of the class **RP**. Show that these properties can be exchanged by the following two:

a) If $w \notin L$, then

$$\Pr_{y \leftarrow \{0,1\}^{p(|w|)}}[N \text{ accepts } w \text{ on the computational branch corresponding to } y] = 0 \ .$$

b) If $w \in L$, then

$$\Pr_{y \leftarrow \{0,1\}^{p(|w|)}}[N \text{ accepts } w \text{ on the computational branch corresponding to } y] \geq 1/2 \ .$$

Present an analogous definition for the class co-**RP**.

Exercise 4:
Recall that $\mathbf{ZPP} = \mathbf{RP} \cap \text{co-}\mathbf{RP}$. Show that if $L \in \mathbf{ZPP}$ then there exist a probabilistic algorithm with expected polynomial time which outputs 1 on input $w \in L$ and outputs 0 on input $w \notin L$.

Exercise 5:
For every δ, $0 < \delta < 1$ define the class $\mathbf{RP}(\delta)$ as the class of languages L for which there exist a polynomial $p : \mathbb{N} \to \mathbb{N}$ and a p-balanced NTM N with following properties:

a) If $w \in L$, then at least $\delta \cdot 2^{p(|w|)}$ of the computation branches of N on input w accept.

b) If $w \notin L$, then all computation branches of N on input w reject.

Note that $\mathbf{RP} = \mathbf{RP}[1/2]$ according to the definition of **RP**. Show that for every δ as above it holds $\mathbf{RP} = \mathbf{RP}[\delta]$.