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Complexity Theory

SS 2016

Class Handout 12

Exercise 1:
Show that $\mathbf{NP}(\mathbf{NP}) = \mathbf{NP}^{SAT}$.

Exercise 2:
Recall that we proved on Class Handout 7 that if $\mathbf{P} = \mathbf{NP}$, then $MINBOOL \in \mathbf{P}$. Re-prove this fact using your knowledge about the polynomial time hierarchy.

Exercise 3:
Find a place for the language

$$L = SAT \times \overline{SAT}$$

in the polynomial time hierarchy. Show that you cannot place it lower: if L is in any of the lower classes, then $\mathbf{NP} = \text{co-}\mathbf{NP}$.

Exercise 4:
We want to give an intuition for the claim that diagonalization is oracle-agnostic. Consider the following diagonalization argument proving that there are languages that are not recursively enumerable:
Let $M_1, M_2, \ldots$ be an enumeration of all DTMs.
We construct language A as follows: Let $A_0 = \emptyset$. To define A_i, assume that A_j for $j < i$ has already been defined properly. If M_i accepts 1^i, set $A_i := A_{i-1}$. Otherwise, set $A_i := A_{i-1} \cup \{1^i\}$. Finally, set $A := \cup_{i \geq 1} A_i$.

a) Finish the proof by arguing that A is not Turing-recognizable.

b) Show that there is an oracle L such that A can be recognized by some OTM M^L.

c) Show that for every oracle L, there is a language A' that is not recognized by any OTM M^L.

Note that this also immediately implies that there are still undecidable problems in a relativized world where the halting problem is decidable.