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Complexity Theory

SS 2016

Homework 5

Exercise 1 (4 points)**:**
As usual, for functions $f, g : \mathbb{N} \to \mathbb{N}$, we write $f(\mathcal{O}(g(n))) := \{f \circ h \mid h \in \mathcal{O}(g(n))\}$. Show that

- $\mathcal{O}(2^n) \neq 2^{\mathcal{O}(n)}$ (inequality of sets).
- $2^{\mathcal{O}(f(n))} = k^{\mathcal{O}(f(n))}$ for any $f : \mathbb{N} \to \mathbb{N}$ and $k \in \mathbb{N}, k > 1$ (equality of sets).

Exercise 2 (4 points)**:**
Use the characterization of **NP** from Theorem 3.4 and give concrete k, A such that

$$SAT = \{x \in \Sigma^* \mid \exists z \in \{0,1\}^{|x|^k} : \ (x, z) \in A\}$$

In particular, you should argue why your solution is correct and that $A \in \mathbf{P}$.

Exercise 3 (4 points)**:**
Use the characterization of co$-$**NP** from Theorem 3.4 and give concrete k, B such that

$$TAUT = \{x \in \Sigma^* \mid \forall z \in \{0,1\}^{|x|^k} : \ (x, z) \in B\}$$

In particular, you should argue why your solution is correct and that $B \in \mathbf{P}$.