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Complexity Theory

SS 2016

Homework 2

Exercise 1 (8 points):
Consider the following description of an algorithm that supposedly decides $\overline{SAT} = \{\langle\phi\rangle \mid \phi \text{ is a Boolean formula and } \phi \notin SAT\}$ in nondeterministic polynomial time:

"On input a Boolean formula ϕ over variables $X_1, \ldots, X_n$:
1. Nondeterministically generate assignments $(x_1, \ldots, x_n) \in \{0,1\}^n$.
2. If $\phi(x_1, \ldots, x_n) = 1$, then reject.
3. If we *never* reject in Step 2, then accept."

a) Why can this description not be implemented on a polynomial time NTM?

b) As it turns out, the issue above cannot be easily fixed. More specifically, we don't know whether or not $\overline{SAT} \in$ **NP**. Prove that $\overline{SAT} \in$ **PSPACE**.

Exercise 2 (8 points):
A *steady sequence* over a set $S \subseteq \Sigma^*$ is a sequence $(s_1, \ldots, s_k) \in S^k$ of strings such that for all i ($1 \leq i < k$), it holds that $|s_i| = |s_{i+1}|$ and that s_i, s_{i+1} differ at exactly one position. For example, (fear, bear, beer, deer, deed, feed, feet) is a steady sequence over the set of English words.
We define

$$STEADY_{\text{DFA}} := \{\langle M, s, t\rangle \mid M \text{ is a DFA, } s, t \in \Sigma^* \text{ and there exists a steady sequence over } L(M) \text{ starting in } s \text{ and ending in } t\}.$$

a) Prove that $STEADY_{\text{DFA}} \in$ **NPSPACE**.

Exercise 3 (8 points):
Let $G = (V, E)$ be a directed graph and $b_0 \in V$ a node. Consider the following game between Player 0 and Player 1. In round $i \geq 0$, Player $[i \mod 2]$ must choose a neighbor node b_{i+1} of b_i such that $b_{i+1} \notin \{b_0, \ldots, b_i\}$. If there is no such b_{i+1}, then the game stops and Player $[i+1 \mod 2]$ wins.
We say that the starting player (Player 0) has a winning strategy if there is a way of playing his turns such that he always wins (no matter which choices the other player makes).
We define

$$GG = \{\langle G, b\rangle \mid \text{The starting player has a winning strategy in the game based on graph } G \text{ with starting node } b\}$$

a) Prove that $GG \in$ **PSPACE**.