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# Complexity Theory

SS 2016

Homework 12

**Exercise 1** (6 points)**:**
Prove Theorem 6.4 part 2, namely:

$$\mathbf{NP}^{TQBF} \subseteq \mathbf{NPSPACE} \subseteq \mathbf{PSPACE} \subseteq \mathbf{P}^{TQBF}.$$

**Exercise 2** (8 points)**:**
Prove Lemma 7.3 by showing that for all $k \in \mathbb{N}_0$

$$\Delta_k = \text{co-}\Delta_k \subseteq \Sigma_k \cap \Pi_k \subseteq \Sigma_k \cup \Pi_k \subseteq \Delta_{k+1} \subseteq \Sigma_{k+1}.$$

**Exercise 3** (10 points)**:**
In the following, we prove that there is an oracle $A$ such that $\mathbf{NP}^A \neq \text{co-}(\mathbf{NP}^A)$.

a) For any language $A$, let $L_A = \{1^n \mid \exists x \in A : \ |x| = n\}$ like in the lecture. Show that for any $A$, it holds that $\overline{L_A} \in \text{co-}(\mathbf{NP}^A)$.

b) It remains to show that there is an $A$ such that $\overline{L_A} \notin \mathbf{NP}^A$.
Consider the construction of $A$ in the lecture, which needs to be adapted for our proof. We give the following hints:

- The general approach and the definition of $e_i, n_i$ can stay the same.
- If there is an accepting computation path for $M_i^?$ on input $1^{n_i}$, we need to ensure that $1^{n_i} \notin \overline{L_A}$ by including at least one $w_i$ of the correct length in $A_i$.
- When including that particular $w_i$, we need to make sure that the oracle answers on the accepting computation path are unaffected. How should you define $X_i$?
- If there is no accepting computation path for $M_i^?$ on input $1^{n_i}$, we need to ensure that $1^{n_i} \in \overline{L_A}$ by excluding all $x$ of length $n_i$ from $A$ (i.e. including them in $\tilde{A}_i$).

If suffices to give the construction of $A$. You do not have to formally prove that $\overline{L_A} \notin \mathbf{NP}^A$.