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Complexity Theory

SS 2016

Homework 10

Exercise 1 (12 points)**:**
Let $f : \{0,1\}^* \to \{0,1\}^*$. We say that f is *nondeterministically* log space computable if there is a log space NTM N with input and output tape such that

- N accepts all inputs.
- On input $x \in \{0,1\}^*$, all reachable accepting configurations of N have $\triangleright f(x)$ on the output tape.

Consider the following tasks (related to the proof that $\mathbf{NL} = \text{co-}\mathbf{NL}$).

a) For a directed graph G and a node s, let $f(G, s)$ be the number of nodes in G reachable from s. For malformed input, $f(G, s) = 0$.
Show that f is nondeterministically log space computable. Give an algorithm computing f and argue *briefly* that it fulfills the bullet points of the definition above.

b) Let $L = \{\langle G, s, t, c\rangle \mid G = (V, E), s, t \in V, c \in \mathbb{N}$ and there is a subset of at least c nodes in $V \setminus \{t\}$ that are reachable from $s\}$.
Argue that $L \in \mathbf{NL}$. Give an algorithm and argue that it indeed decides L.

c) What is the relation between f, L, and $\overline{PATH}$?

d) Deduce that $\overline{PATH} \in \mathbf{NL}$ by describing an algorithm referencing f and L. Argue that there is an accepting computation path if and only if the input is in $\overline{PATH}$.

General remark: you should give high level arguments. Please do not argue about the code line-by-line.

Exercise 2 (8 points)**:**
We say a language $L \subseteq \Sigma^*$ is $\mathbf{P}$-complete, if

- $L \in \mathbf{P}$.
- Every $L' \in \mathbf{P}$ is *log space reducible* to L.

(Note that this is one possible way to define $\mathbf{P}$-completeness. Another definition relates more naturally to parallel computation).
Show that

a) If L is $\mathbf{P}$-complete and $L \in \mathbf{L}$, then $\mathbf{L} = \mathbf{P}$.

b) $\{\langle M, x, 1^t\rangle \mid$ the Turing machine M accepts x within t steps$\}$ is $\mathbf{P}$-complete.