

## Cryptography - Provable Security

SS 2016

Handout 7

*Exercises marked (\*) and (\*\*) will be checked by tutors.*

*We encourage submissions of solutions by small groups of up to four students.*

### Exercise 1:

Prove that if there exists a pseudorandom generator, then there exists a 1-way function (Theorem 5.18 from the lecture).

**Hint:** Prove that a PRG with expansion factor  $2n$  is a 1-way function.

### Exercise 2 (4 points):

(\*\*) Consider Theorem 7.5 from the lecture and the corresponding multiple messages eavesdropping game  $\text{PubK}_{A,\Pi}^{\text{mult}}(n)$ . Extend at first the experiment to the CCA setting in an appropriate way. Next, assume that the underlying public-key encryption scheme  $\Pi$  is CCA-secure. Does it necessarily have multiple indistinguishable encryptions under a chosen ciphertext attack? Prove your answer formally.

### Exercise 3:

Consider the hybrid encryption scheme defined in the lecture. Let  $\Pi$  be a CCA-secure public-key encryption scheme (define an appropriate experiment for this) and  $\Pi'$  be a CCA-secure private-key encryption scheme. Is the hybrid construction  $\Pi^{\text{hyb}}$  instantiated using  $\Pi$  and  $\Pi'$  also CCA-secure? Prove your answer formally. I. e., does an analogue for Theorem 7.11 hold for CCA security?

### Exercise 4 (4 points):

(\*\*) Let  $G = G_0 \times G_1$  be a pseudorandom generator with expansion factor  $2n$  such that for all  $x \in \{0, 1\}^n$

$$G(x) = (G_0(x) \| G_1(x)) \quad \text{and} \quad |x| = |G_0(x)| = |G_1(x)|.$$

Prove that

$$\tilde{G}(x) = (G_0(G_0(x)) \| G_0(G_1(x)) \| G_1(G_0(x)) \| G_1(G_1(x)))$$

is a pseudorandom generator with expansion factor  $4n$ .